

For each of the systems below, find

- the complementary function
- the particular integral
- the full general solution

and sketch the response.

Find the damping ratio and, if the system is under-damped ($\zeta < 1$), the frequency and amplitude of oscillation.

$$(1) \quad \ddot{x} + \dot{x} + x = 0 \quad x(0) = 1 \quad \dot{x}(0) = 0$$

$$(2) \quad \ddot{y} + 2\dot{y} + y = 0 \quad y(0) = 1 \quad \dot{y}(0) = 0$$

$$(3) \quad \ddot{z} + 3\dot{z} + z = 0 \quad z(0) = 1 \quad \dot{z}(0) = 0$$

$$(4) \quad \dot{p} + p = 0 \quad p(0) = 1 \quad \dot{p}(0) = 0$$

$$(5) \quad \ddot{q} + q = 0 \quad q(0) = 1 \quad \dot{q}(0) = 0$$

$$(6) \quad \ddot{\theta} + 2\dot{\theta} + \theta = t \quad \theta(0) = 0 \quad \dot{\theta}(0) = 0$$

$$(7) \quad \ddot{\phi} + 3\dot{\phi} + \phi = e^{-3t} \quad \phi(0) = 2 \quad \dot{\phi}(0) = 0$$

$$(8) \quad \ddot{\psi} + 4\dot{\psi} + \psi = \cos 2t \quad \psi(0) = 0 \quad \dot{\psi}(0) = 0$$